Arithmetic for First Graders Lacking Number Concepts

Photograph by Judith Rummelsburg; all rights reserved

Among the first graders who came to our Title-I school one year, we found twenty-six children who had no understanding of number concepts. In the assessment at the beginning of the school year, these children could not conserve number with eight counters. They could count out four chips, but when we hid some of the chips and asked, "How many am I hiding?" the children gave random answers, such as, "Ten." Our challenge was that we were required by law to teach an hour of arithmetic to these children every day despite the fact that they had not yet developed an understanding of number concepts.

Such first graders are usually given exercises in counting objects, making one-to-one correspondences, and filling out activity sheets with problems such as 2 + 3. However, on the basis of Piaget's research and theory (Piaget 1954; Piaget and Szeminska 1952; Inhelder and Piaget 1964), we decided to test a hypothesis. It takes five to six years for most children to construct meaning of number concepts, according to Piaget, and all children start in infancy to build a cognitive foundation for number. One of us was familiar with preschool education and knew that physical-knowledge activities are especially good for three- and four-year-olds' development of this foundation (Kamii and DeVries 1993). We decided to test the hypothesis that these physical-knowledge activities are good for slow-to-develop first graders to build a cognitive foundation for number.

By Constance Kamii and Judith Rummelsburg

Constance Kamii, ckamii@uab.edu, teaches early childhood education at the University of Alabama at Birmingham. Her research interest is in developing better ways of teaching based on Piaget's theory. Judith Rummelsburg, judymccall @sbcglobal.net, teaches a combination second-third grade at Shore Acres Elementary School in Bay Point, CA. She is interested in applying Piaget's theory in a classroom setting.

Physical-Knowledge Activities

Physical-knowledge activities are those in which children act on objects physically and mentally to produce a desired effect. Examples of physical-knowledge activities are the Bowling game, Pick-Up Sticks, the Balance game, and Jenga. In the Bowling game, children arrange eight to ten "pins" (empty plastic beverage bottles) and roll a tennis ball to knock over as many as possible. In Pick-Up Sticks, they drop fifteen plastic sticks in a heap on the floor and try to pick up one stick at a time without moving any other stick. In the Balance game (see **fig. 1**), students put a paper plate on an empty plastic bottle and take turns adding one Unifix cube at a time to the plate without making it fall. Jenga has them take turns pulling out one block at a time while trying to keep the tower of blocks from falling (see **fig. 2**). These activities encourage children to think deeply, and they can tell immediately whether or not they are successful. If they are unsuccessful, they are motivated to figure out what to do differently the next time. Children build a foundation for number by thinking about quantity, according to Piaget, and physical-knowledge activities help children think in such a manner.

Figure 1

In the Balance game, students put a paper plate on an empty plastic beverage bottle and take turns adding one Unifix cube at a time to the plate without making it fall.

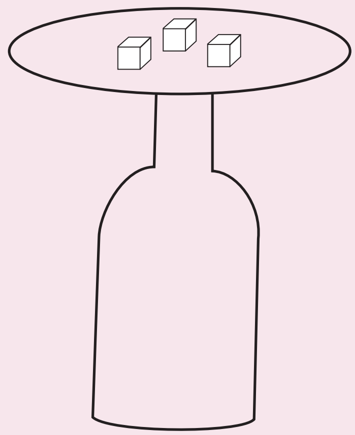


Figure 2

In Jenga, students take turns pulling out one block at a time while trying to keep the tower from falling.

Danijel Micka/iStockphoto.com

How children build this foundation is explained later in this article; we first describe what we did in the classrooms and the outcome of this experiment. Two teachers divided a class of twenty-six first graders into two classes of thirteen each and gave them physical-knowledge activities every day during the mathematics hour. The students in each class were further organized into pairs to maximize their depth of thinking.

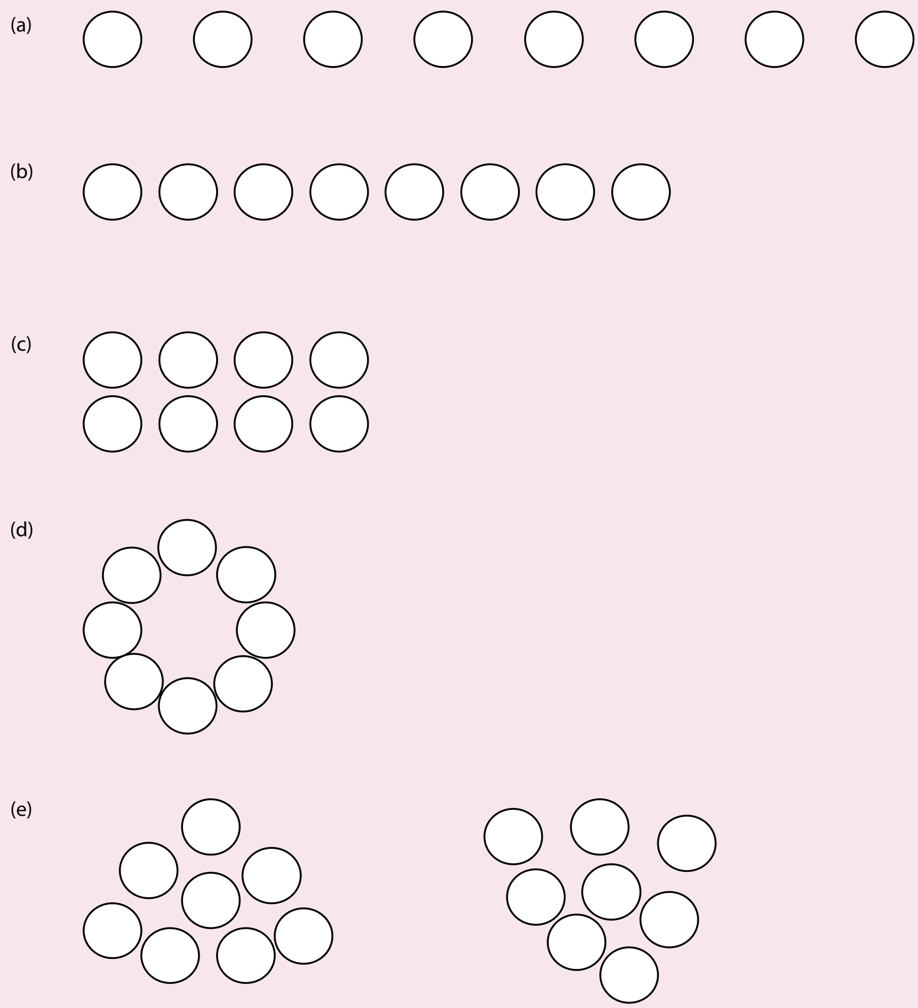
The children truly enjoyed these activities and carefully studied how the changes in their actions produced different results. In the Bowling game, for example, they arranged eight "pins" in a line with too much space between them (see **fig. 3a**), bunched them close together (see **fig. 3b**), arranged them in two lines (see **fig. 3c**), into a circle (see **fig. 3d**), and into two different triangle shapes (see **fig. 3e**). The students were also motivated to count the pins they knocked over, and we could tell from their concentration and joy that these activities were at the right level for our first graders.

As teachers, we were careful to avoid making any suggestions to the students when they were unsuccessful. Instead, we asked questions, such as, "What could you do differently next time?" The purpose of these activities was to encourage the students to think in their own ways—to figure out how a desired effect can be produced. We also worked hard on their social behavior and interaction, which often started at the level of three-year-olds.

After the winter break, we began to assess the children's "readiness" for arithmetic by playing the

Figure 3

The hands-on aspects of the Bowling game captivated the first graders' attention and motivated them to place the bowling "pins" in different arrangements for different outcomes.



Skip O'Donnell/iStockphoto.com

Piggy Bank game with each child. This is a game using a total of forty cards, ten of each card type: showing one, two, three, or four dots. The object of the game is to find two cards that equal five when their numbers of dots are added together. The cards are dealt to two players, who keep their respective stacks facedown. The players take turns turning over the top card on their stacks. If the student cannot make a sum of five with the card she has just turned over and one in the discard pile, the card that has just been turned over must also be discarded, faceup, in the middle of the table. (Little children love to take the first turn, but the first turn in this case—with only one card faceup—will never result in a sum of five!) When a player is able to make a sum of five with a pair of cards, the player can keep the cards in her "piggy bank." The winner is the child who has more cards banked at the end of the game.

Students who played the Piggy Bank game easily and eagerly went on to arithmetic with word problems and the mathematics games (described in Kamii 2000, chaps. 9 and 11). Students who found the Piggy Bank game too challenging continued to play physical-knowledge games and other strategy games (described in Kamii 2000, chap. 10). Thus, the twenty-six first graders were asked to deal with arithmetic *only* when they showed developmental readiness. By the end of February, most of the children were solving word problems and playing mathematics games. Moreover, all the teachers in the school used teacher-created word problems instead of a textbook, and mathematics games rather than activity worksheets.

Evaluating the Results

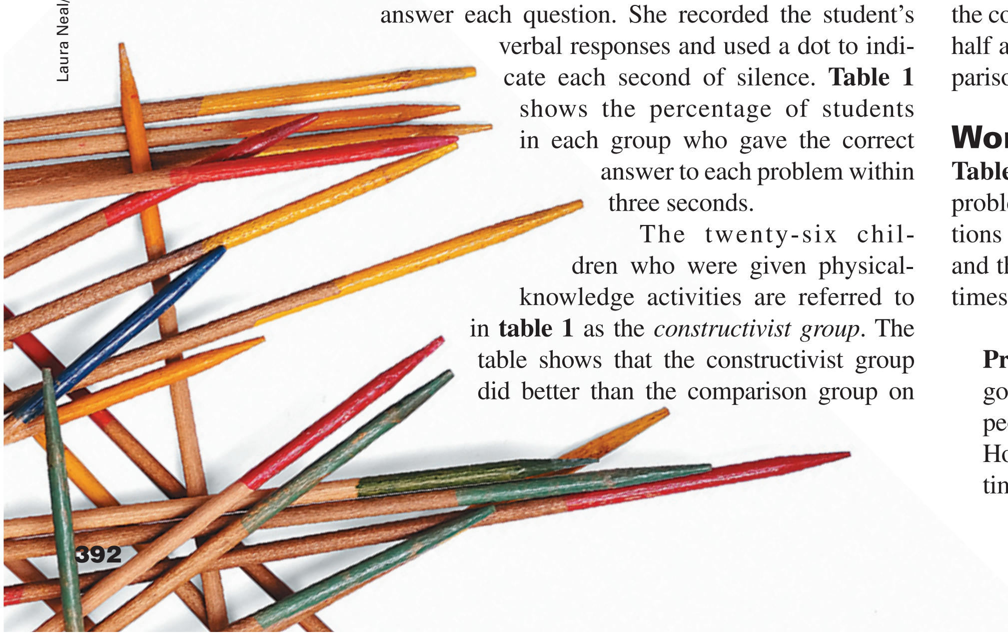
In May, we gave a posttest to our twenty-six students, as well as to a comparison group of twenty first graders in a nearby school. The two groups were very similar at the beginning of the school year, with averages of 78.6 and 79.38, respectively, on a September pretest given to all first graders in the District's Title-I schools. The pretest was an orally presented multiple-choice group test published by Houghton Mifflin (2002).

The twenty low-performing first graders in the comparison group were given many worksheets throughout the year with problems similar to those on **table 1**. The students in the comparison group answered these questions with the aid of counters or their fingers. Their teachers sometimes provided other activities recommended by two well-known, respected authors.

Mental arithmetic

Our posttest consisted of mental arithmetic and four word problems. The data on mental arithmetic are presented in **table 1**. These data were collected in individual interviews. The child and the interviewer each had a sheet of paper showing problems (see **table 1**, column 1). The student was given a ruler and asked to place it under the first problem (2 + 2), and then, after giving each answer, to slide it down to the next problem. The interviewer could tell, therefore, how many seconds it took each child to answer each question. She recorded the student's verbal responses and used a dot to indicate each second of silence. **Table 1** shows the percentage of students in each group who gave the correct answer to each problem within three seconds.

The twenty-six children who were given physical-knowledge activities are referred to in **table 1** as the *constructivist group*. The table shows that the constructivist group did better than the comparison group on every problem except 3 + 3. The table also shows that the differences between the two groups were statistically significant on eight of the seventeen problems. In other words, the twenty-six children in the constructivist group did not do any arithmetic for half a year but ended up doing better than the comparison group in mental arithmetic.



Laura Neal/iStockphoto.com

Table 1

First Graders' Correct Answers within Three Seconds in Mental Arithmetic (in Percent)

	Constructivist Group	Comparison Group	Difference	Statistical Significance
	n = 26	n = 20		
2 + 2	100	90	10	n.s.
5 + 5	92	90	2	n.s.
3 + 3	77	85	-8	n.s.
4 + 1	88	65	23	.05
1 + 5	88	70	18	n.s.
4 + 4	88	65	23	.05
2 + 3	81	40	41	.01
4 + 2	58	25	33	.05
6 + 6	50	40	10	n.s.
5 + 3	58	35	23	n.s.
8 + 2	69	45	24	.05
2 + 5	62	40	22	n.s.
4 + 5	42	30	12	n.s.
5 + 6	24	5	19	.05
3 + 4	38	15	23	.05
3 + 6	38	10	28	.05
4 + 6	35	20	15	n.s.

Word problems

Table 2 shows the findings from the four word problems we gave. Each of the following four questions was photocopied on a separate sheet of paper, and the interviewer read them to the child as many times as requested by the child.

Problem 1. (Line) People started to get in line to go to lunch. I was standing in line and counted 3 people in front of me and 6 people in back of me. How many people were in line altogether at that time?

Table 2

Two Groups' Responses to Word Problems (in Percent)

	Constructivist Group	Comparison Group	Difference	Statistical Significance
	n = 26	*n* = 20		
1. Line (10)	8	0	8	n.s.
2. Crackers (8)	19	5	14	n.s.
3. Cookies (2)	50	0	50	.001
4. Candy (6)	73	25	48	.001

Problem 2. (Crackers) I am getting soup ready for 4 people. I have 4 bowls. If I want to put 2 crackers in each bowl, how many crackers do I need?

Problem 3. (Cookies) There are 3 children. There are 6 cookies for them to share. How many cookies will each child get?

Problem 4. (Candy) Let's pretend that I had 12 pieces of candy. If I gave 2 pieces to my mother, 2 pieces to my father, and 2 pieces to my sister, how many pieces would I have left?

Pencils were provided, and the students were told they could use them to draw or write anything that might help to figure out the answer.

The numbers involved in these questions were small and easy for the children to work with, but the logic was not. The first question required the inclusion of the self in the line. The second question was a multiplication problem, which the children could solve by using repeated addition. The third question was a division problem, which first graders could also solve with addition. The last problem required subtracting 2 three times.

Table 2 shows that the constructivist group did better than the comparison group on all four of the word problems, and that the differences were highly significant on two of them. The third problem about cookies was solved by half of the constructivist group and none of the comparison group. The fourth problem about candy was solved by 73 percent of the constructivist group and only 25 percent of the comparison group.

Why Did the Constructivist Group Do Better?

To understand why the constructivist group performed better, we must review the fundamental distinction Piaget made among three kinds of knowledge according to their ultimate sources—physical, logico-mathematical, and social (conventional) knowledge. *Physical knowledge* is knowledge of objects in the external world. The color of the counters and the fact that they are made of plastic are examples of physical knowledge. The fact that they do not roll away like marbles do is also an example of physical knowledge. An example of *social (conventional) knowledge* is the fact that counters are called *counters* or *chips*. Words phrases such as *one-two-three* and *uno-dos-tres* are also examples of social knowledge. Physical knowledge has a source in objects, and social knowledge originates in conventions that people make.

Whereas physical and social knowledge have sources outside the individual, *logico-mathematical* knowledge consists of mental relationships that originate inside each individual's head. If we are shown a red counter and a blue one, for example, we can say that they are different. In this situation, the difference originates not in the objects but in each individual who thinks about the counters as "different." The proof is that, if the individual decides to ignore color, the counters can become "similar" or "the same" for him. On the other hand, if the person decides to think numerically about the two counters, the counters can become "two." The ultimate source of logico-mathematical knowledge is in each individual who puts objects into mental relationships. The two counters are observable (physical knowledge), but "two" is not. "Two" is a mental relationship that each individual must construct in his mind, and each child goes on to construct "eight," "ten," "twenty," and so on.

Often cognitive development is believed—incorrectly—to occur through biological maturation, but Piaget emphatically said it takes place through mental action, or the child's thinking. Children who are mentally active develop faster than those who are passive. This is why we gave physical-knowledge activities to our low-performing first graders, to encourage them to think logico-mathematically. To figure out how to act on objects, they made many logico-mathematical relationships. In the Pick-Up Sticks game, for example, they created categories, such as "the sticks that are not touching any other stick" and "all the others." They also seriated the sticks into "those that are easiest," "those that are a little harder," and "those that are the hardest" to pick up. They coordinated this seriation with temporal relationships when they decided to pick up the easiest sticks first, the next easiest second, and so on. Among the spatial relationships involved in this coordination are "on top of" and "at the bottom." At the end of the game, children had to make numerical relationships to know who won.

When we advanced to arithmetic, we gave mathematics games to our students rather than activity worksheets because (a) games encourage logico-mathematical thinking, and (b) children love games. In the Piggy Bank game, for instance, children were free to think about 4 + 1, 3 + 2, 2 + 3, and 1 + 4 because they did not have to write anything. When they are given activity sheets, many become preoccupied with such things as how to write a number *5* differently than a letter *S*. Numerals and equations belong to social knowledge, which is the most superficial part of arithmetic and should not interfere with children's acquisition of logico-mathematical knowledge. When the logico-mathematical relationship among 3, 2, and 5 becomes second nature to them, the social knowledge of numerals and equations becomes easy.

We continue to encourage logico-mathematical thinking after our students show a readiness for arithmetic. Our mathematics time usually begins with a word problem. We do not use any textbook because textbooks stifle children's ability to do their own thinking. Textbooks typically introduce addition and then proceed to subtraction. By contrast, we give problems throughout the year that would traditionally be called *addition*, *subtraction*, *multiplication*, and *division* because research has shown that kindergartners and first graders can solve multiplication and division problems (Carpenter et al. 1993; Kamii 2000). For instance, to know how six cookies can be divided among three children, first graders draw three big circles (for three children) and six small ones (for six cookies), and cross out each cookie as they give one to each of the three children.

According to Piaget, logico-mathematical knowledge begins to develop in infancy (1954), on the first day of life. Therefore, no matter how low a child's level of performance is at whatever grade level, he is always somewhere along the developmental continuum. In the case of our twenty-six students, we thought that they were at the level of most three- and four-year-olds, and we gave them physical-knowledge activities that encouraged them to think. The children quickly strengthened their foundation for number concepts and engaged in arithmetic during the second half of the year. With a good cognitive foundation, they learned arithmetic more quickly than the comparison group, which was given traditional mathematics activities that required low-level thinking (like mechanical counting).

However, encouraging first graders to do their own thinking during first grade is not sufficient. Traditional second-grade instruction teaches "carrying" and "borrowing" that make most children give up their own thinking (Kamii and Dominick 1998). Second-grade arithmetic is beyond the scope of this article, but we want readers to be aware that low-performing students remain vulnerable to teaching sequences that make no sense to them. For further information about what to do in second grade, third grade, and beyond, refer to Kamii (1994; 2004) and Kamii, Rummelsburg, and Kari (2005).

References

Carpenter, Thomas P., Ellen Ansell, Megan L. Franke, Elizabeth Fennema, and Linda Weisbeck. "Models of Problem Solving: A Study of Kindergarten Children's Problem-Solving Processes," *Journal for Research in Mathematics Education* 24 (1993): 428–41.

Houghton Mifflin. Readiness Test (Grades 1–5: California Standards-Based Assessment). Boston: Houghton Mifflin, 2002.

Inhelder, Barbel, and Jean Piaget. *The Early Growth of Logic in the Child*. New York: Harper and Row, 1964.

Kamii, Constance. *Young Children Continue to Reinvent Arithmetic, 2nd Grade*. 2nd ed. New York: Teachers College Press, 2004.

———. *Young Children Continue to Reinvent Arithmetic, 3rd Grade*. New York: Teachers College Press, 1994.

———. *Young Children Reinvent Arithmetic*. 2nd ed. New York: Teachers College Press, 2000.

Kamii, Constance, and Rheta DeVries. *Physical Knowledge in Preschool Education*. New York: Teachers College Press, 1993.

Kamii, Constance, and Ann Dominick. "The Harmful Effects of Algorithms in Grades 1–4." In *The Teaching and Learning of Algorithms in School Mathematics*. 1998 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Lorna J. Morrow and Margaret J. Kenney, pp. 130–40. Reston, VA: NCTM, 1998.

Kamii, Constance, Judith Rummelsburg, and Amy Kari. "Teaching Arithmetic to Low-Performing, Low-SES First Graders." *Journal of Mathematical Behavior* 24 (2005): 39–50.

Piaget, Jean. *The Construction of Reality in the Child*. New York: Basic Books, 1954.

Piaget, Jean, and Alina Szeminska. *The Child's Conception of Number*. London: Routledge and Kegan Paul, 1952. ▲